

Homework 2

Due 1/20/2011

1. [20 points] Kittel 2.1
2. [10 points] Kittel 2.2
3. [10 points] In order to satisfy the Bragg condition, the wavelength of a quantum particle that diffracts off of the crystal needs to be on the order of, or smaller than, the spacing of lattice planes. Since the typical lattice spacing of a crystal is a few Å, the wavelength of about 1 Å can be taken as a typical required value. Find corresponding energies in unit of eV (electron volt) when the probing particle is (a) the photon, (b) the electron, and (c) the neutron. (d) Which particle can be described as "thermal" in the sense that its energy is on the order of the room temperature 300 K (Note that, by the equipartition theorem, the thermal energy $\sim k_B T$). You may find useful the "Physical constants" sheet that was distributed in class. [NOTE: In this problem, the "energy" is to be interpreted as only the kinetic energy portion, without including the energy mc^2 where m is the rest mass.]
4. [10 points] Kittel 2.4
5. [10 points; extra credit] Consider a D -dimensional Bravais lattice, where the dimension D can be any positive integer, generated by D linearly independent vectors $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_D$, as $\vec{R} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + \dots + n_D \vec{a}_D$ with $n_1, \dots, n_D \in \mathbb{Z}$ (the set of integers). Then, the definition of its reciprocal lattice $\{\vec{G}\}$ can be generalized (following what we did in class) as follows. First, find D \vec{a}_i^* vectors that satisfy

$$\vec{a}_i^* \cdot \vec{a}_j = 2\pi \delta_{ij} \quad i, j = 1, \dots, D$$

And span the reciprocal lattice using

$$\vec{G} = m_1 \vec{a}_1^* + m_2 \vec{a}_2^* + \dots + m_D \vec{a}_D^*, \quad m_i \in \mathbb{Z}$$

Let V be the volume of the unit cell of the original Bravais lattice and V^* be the volume of the unit cell of the reciprocal Bravais lattice. Show that

$$VV^* = (2\pi)^D$$

Hint: With \vec{a}_i vectors, form a $D \times D$ matrix. Similarly for \vec{a}_i^* vectors. Use matrix algebra (determinant, the geometrical meaning of the determinant, transpose, multiplication).